Lecture 2. B.Sc-II

2. Equations solvable for y:

If the equation is solvable for y we differentiate the solved from say $y = F(x, p)$, with respect to x.

$$\therefore \frac{dy}{dx} = p = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial p} \cdot \frac{dp}{dx}$$

$$= \phi\left(x, p, \frac{dp}{dx}\right) \quad \text{(say)}$$

We solve $p = \phi\left(x, p, \frac{dp}{dx}\right)$ to obtain $\psi(x, p, c) = 0$.
The primitive is obtained by eliminating p between $y = F(x, p)$ & $\psi(x, p, c) = 0$.
Sometimes we write down the solution by expressing x & y separately as functions of p, a parameter.

Example ① Solve $y = px + p^2 x$ — ①

Solution: → $y = F(x,p)$

Differentiating w.r.to x, we get

$$p = p + x\frac{dp}{dx} + p^2 + 2xp\frac{dp}{dx}$$

$$\Rightarrow -p^2 = x(1+2p)\frac{dp}{dx}$$

$$\Rightarrow \frac{dx}{x} + \frac{1+2p}{p^2}\,dp = 0$$

Integrating, we have.

$$\log x + \log c = \frac{1}{p} - 2\log p$$

$$\Rightarrow \quad cxp^2 = e^{1/p}$$

$$\therefore \quad x = \frac{e^{1/p}}{cp^2} \quad \Rightarrow y = x(p+p^2)$$

$$= \frac{e^{1/p}}{cp^2}(p+p^2)$$

constitute the parametric solution of the given equation (p is the parameter)

<u>Example ②</u>: $\rightarrow$ $y + px = p^2x^4$. — ①

<u>Solution</u>: $\rightarrow$ Differentiating (1) w.r.to x

$$p + p + x\frac{dp}{dx} = 2px^4\frac{dp}{dx} + p^2 \cdot 4x^3$$

$$\Rightarrow 2p(1-2x^3p) + x\frac{dp}{dx}(1-2x^3p) = 0$$

$$\Rightarrow (1-2x^3p)\left(2p + x\frac{dp}{dx}\right) = 0$$

$$\therefore \quad 2p + x\frac{dp}{dx} = 0 \text{ if } 1-2x^3p \neq 0$$

$$\Rightarrow 2p = -x\frac{dp}{dx}$$

$$\Rightarrow \frac{dx}{-x} = \frac{dp}{2p}$$

Integrating

$$\Rightarrow -\log x + \log c = \frac{1}{2}\log p$$

$$\Rightarrow 2.\log\frac{c}{x} = \log p \qquad \Rightarrow p = \frac{c_1}{x^2}$$

Eliminating p from $p = \frac{c_1}{x^2}$ & the given equation we obtain the complete primitive as

$$y = -\frac{c_1}{x} + c_1^2 \quad \text{or} \quad xy + c_1 = c_1^2 x$$

Ans.

Example ③ :- Solve $y = p + \frac{x}{p}$ — ①

Solution:→ Differentiating ① w.r.to x

Example ④ :- Solve $y = x(p + p^2)$

Example ⑤ :- Solve $y = x + a\tan^{-1} p$